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$$+\frac{2}{9a^4}\int_0^a\int_0^a 2ay\sqrt{[a^2+y^2+z^2]}+(3z^2y+y^3)\log\frac{a+\sqrt{[a^2+y^2+z^2]}}{\sqrt{[y^2+z^2]}}$$

$$+(3az^2+a^3)\log\frac{y+\sqrt{[a^2+y^2+z^2]}}{\sqrt{[a^2+z^2]}}-2z^3\tan^{-1}\left(\frac{ay}{z\sqrt{[a^2+y^2+z^2]}}\right)\Bigg]dy\,dz$$

$$=\frac{1}{36a^4}\int_0^a\Bigg[4a^2y^2-2\pi a^4-2\pi y^4+4y(2a^2+y^2)^{\frac{3}{2}}+2y(a^2+y^2)^{\frac{3}{2}}$$

$$-6y^3\sqrt{[a^2+y^2]}-20\,a^4\tan^{-1}\left(\frac{y}{\sqrt{[2a^2+y^2]}}\right)+4\,y^4\tan^{-1}\left(\frac{y\sqrt{[2a^2+y^2]}}{a^2}\right)$$

$$+4\,a^4\tan^{-1}\left(\frac{\sqrt{[2a^2+y^2]}}{y}\right)+3a^4\log 2+8(5a^3y+3ay^3)\log\frac{a+\sqrt{[2a^2+y^2]}}{\sqrt{[a^2+y^2]}}$$

$$+2a^2(3a^2+4y^2)\log\frac{y+\sqrt{[a^2+y^2]}}{a\sqrt{2}}+8(3a^4-a^2y^2)\log\frac{y+\sqrt{[2a^2+y^2]}}{a\sqrt{2}}\Bigg]dy$$

$$=\frac{a}{1620}[286-186\pi+464\sqrt{2}-372\sqrt{3}+768\log(1+\sqrt{2})+1752\log(1+\sqrt{3})$$

$$-936\log 2]=\tfrac{25}{27}a,\text{ nearly.}$$

MISCELLANEOUS.

166. Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Several equal rectangular boxes are placed in a row with uniform intervals between the boxes and a passageway along one side of the row. Find the least width of the passageway permitting a box to be removed from the row without moving adjacent boxes. This problem arose during the construction of a room for storage batteries.

Solution by the PROPOSER.

Length of box $=PQ=a$; $\angle DAY=\phi=\cos^{-1}(b/c)$. Breadth of box $=PS=b$; $\angle RAY=\theta$. Space between boxes $=AB=c$; $OA=l=c\cot\phi$. Passage in front of boxes $=PW=h$; $OB=m=c\csc\phi$; $AR=n=c\csc\theta-b\cot\theta$.

The complete solution of this problem leads to seven cases corresponding to the possible relations between a, b, c. The desired solution is $h=PW$, and unless otherwise stated; $\phi\neq0$. The general cases are 1°, $a>c\csc\phi$; 2°, $c\csc\phi>a>c$; 3°, $c>a>c\sin\phi$; 4°, $c\sin\phi>a>0$.

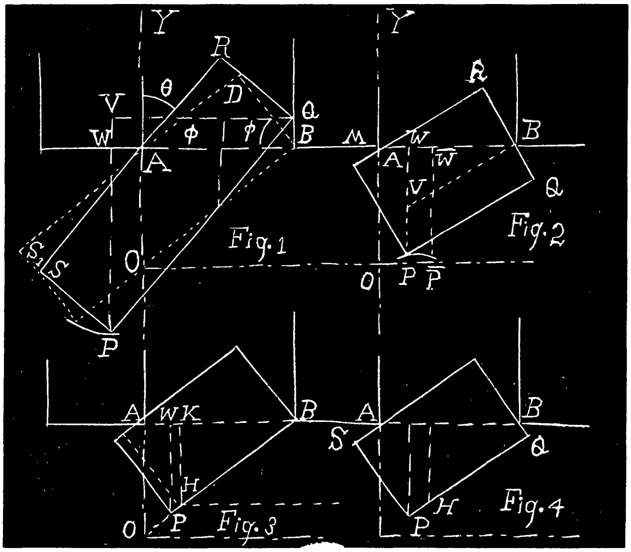
In case 1°, Fig. 1, the box remains partly in the "space" when PW is a maximum. The axes and the auxiliary constant ϕ are determined by the position of the box when Q coincides with B, and the minimum value of y corresponds to the desired value of PW. The path of P is given by

$$x=c-a\sin\theta,\quad y=l+(n-a)\cos\theta-b\sin\theta, \tag{1}$$

in which θ is an auxiliary parameter. The minimum value of y is given by the solution of the transcendental equation

$$c=a\sin^3\theta+b\cos\theta. \tag{2}$$

The instantaneous center of motion of PO is at V, the intersection of the horizontal through Q and the normal to PQ through A. When the box is moving out of the space, V moves toward the left and P toward the right, the minimum y occuring when P crosses the vertical through V. This is the position shown in Fig. 1, from which equation (2) may be verified.



The solution for case 1° is

$$h=l-y=a\cos\theta+b\csc\theta-c\cot\theta, \tag{3}$$

in which θ is the solution of (2).

Fig. 1 shows that h is less than a, so there would be no advantage in withdrawing the box without turning, and then moving it sidewise.

Case 1° is particularly important because the critical point is not at the position BS_1. The limiting case between 1° and 2° occurs when $a=c\csc\phi=OB$, and the solution is then $h=a\cos\phi=ab/c$.

In case 2°, $c\csc\phi>a>c$, the ordinate of P is still decreasing when Q reaches B, as may be shown by the derivative of the second equation in (1) viz.,

$$\frac{dy}{d\theta}=\frac{-1}{\sin^2\theta}(c-a\sin^3\theta-b\cos\theta). \tag{4}$$

Hence the path of P must be studied when SR and QR have sliding contact with A and B, respectively. From Fig. 2,

$$\left.\begin{aligned} x&=b\cos\theta-a\sin\theta+c\sin^2\theta,\\ y&=l-a\cos\theta-b\sin\theta+c\sin\theta\cos\theta, \end{aligned}\right\} \tag{5}$$

$$dy/d\theta = a\sin\theta - c\sin^2\theta + c\cos^2\theta - b\cos\theta = MA - BN. \quad (6)$$

Thus when Q has passed B, the ordinate of P changes and b becomes an increasing one. When a is a little greater than c, P may be in the same vertical as V, the instantaneous center, as seen in Fig. 2, where it is evident that (6) vanishes. This fact does not lead to a solution because P is here at a maximum, so that the provisional solution comes when Q coincides with B, *i. e.*, $\overline{PW}$, Fig. 2.

On PQ let $HQ = \bar{a}$ be laid off so that

$$c > \bar{a} > c\sin\phi, \quad (7)$$

then from Fig. 3,

$$HK = \bar{a}b/c, \quad (8)$$

and it is to be noticed that a further change of position, as shown in Fig. 4, gives by computation, the same value for HK. At the same time (6) shows that during this motion from Fig. 3 to Fig. 4, the ordinate of P increases and then decreases. Since the slope of PQ is less in the latter position, it follows that the ordinate of P is greater in Fig. 4.

If instead of the restriction in (7), $\bar{a}$ be so chosen that

$$c\csc\phi > \bar{a} > c, \quad (9)$$

it follows from (6), since MA does not vanish when R reaches B, that the ordinate of P is still decreasing, hence from (8), with a replacing $\bar{a}$ so as to apply to case 2°,

$$h = PW = ab/c. \quad (10)$$

Thus (10) is the solution for case 2° and gives a value greater than b. In the limiting case between 2° and 3°, *i. e.*, $a = c$, the value is precisely b.

For case 3°, $c > a > c\sin\phi$, Fig. 4 is available if the left boundary is AH instead of SP, so that (8) gives

$$h = HK = ab/c,$$

a result less than b. But a rotation about A must follow, making AH vertical, and now the solution is $h = b$. The limiting case between 3° and 4°, *i. e.*, $a = c\sin\phi$, is that in which the box can just rotate in the space. This case may be considered with case 4°, $c\sin\phi > a$, and the joint solution is $h = b$, as complete rotation is possible.

In the last and trivial case, where $\phi = 0$, b is equal to c, and the solution is $h = a$, because the box cannot be rotated in the space, but must be withdrawn by translation.